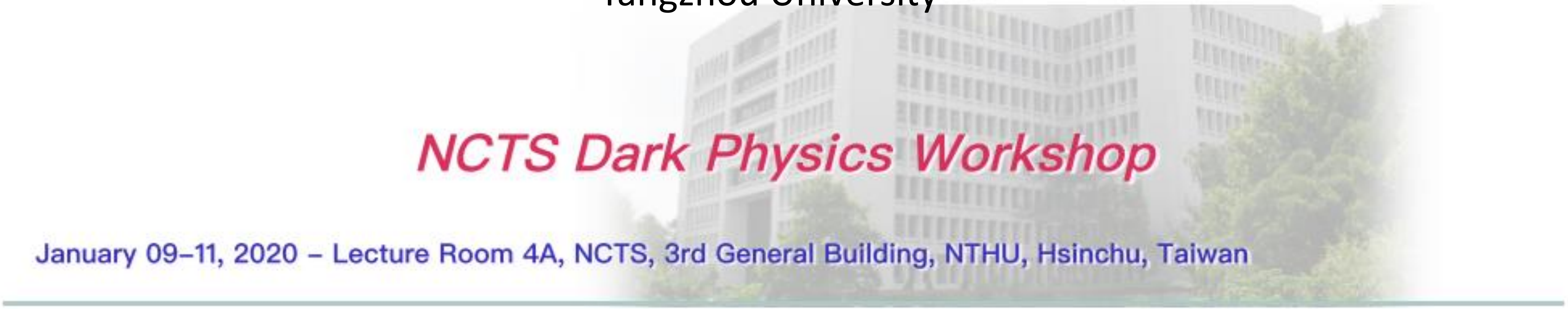
YANGZHOU UNIVERSITY
SINCE 1902

The state of the Schwinger particle pair

De-Chang Dai

Yangzhou University



Main contents

- **Introduction of entanglement**
- **The state of a Schwinger's pair is not necessarily a maximally entangled Bell's state**

1. A new wormhole solution in de Sitter space, D.C Dai, D. Minic, D. Stojkovic, PRD, 98, 124026

2. State of a particle pair produced by the Schwinger effect is not necessarily a maximally entangled Bell state, D.C Dai, PRD 100, 045015

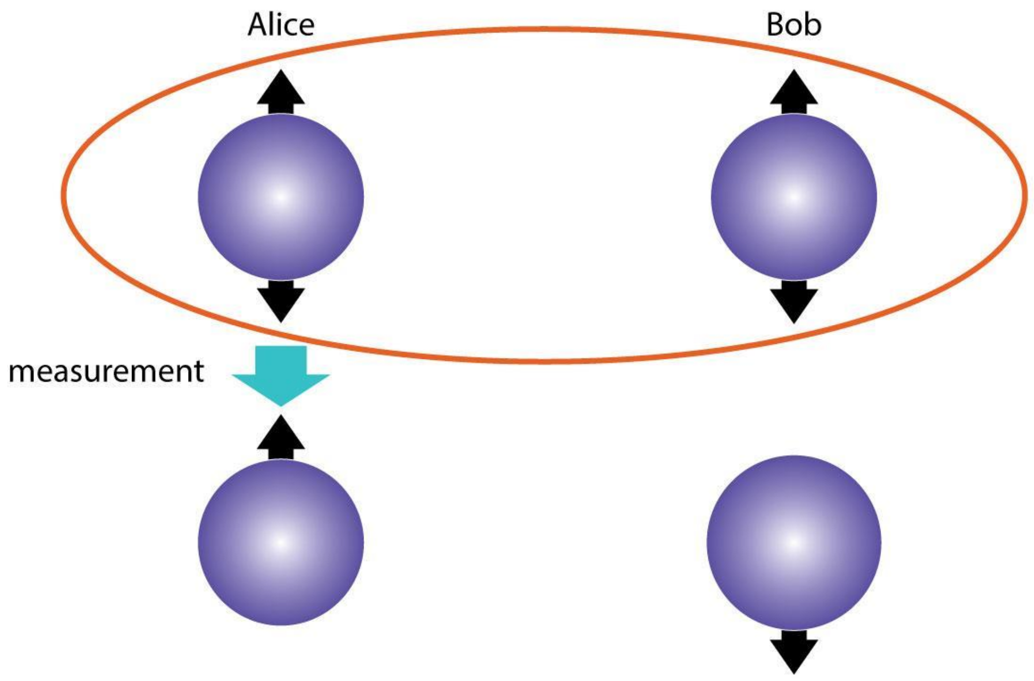
Einstein–Podolsky–Rosen paradox

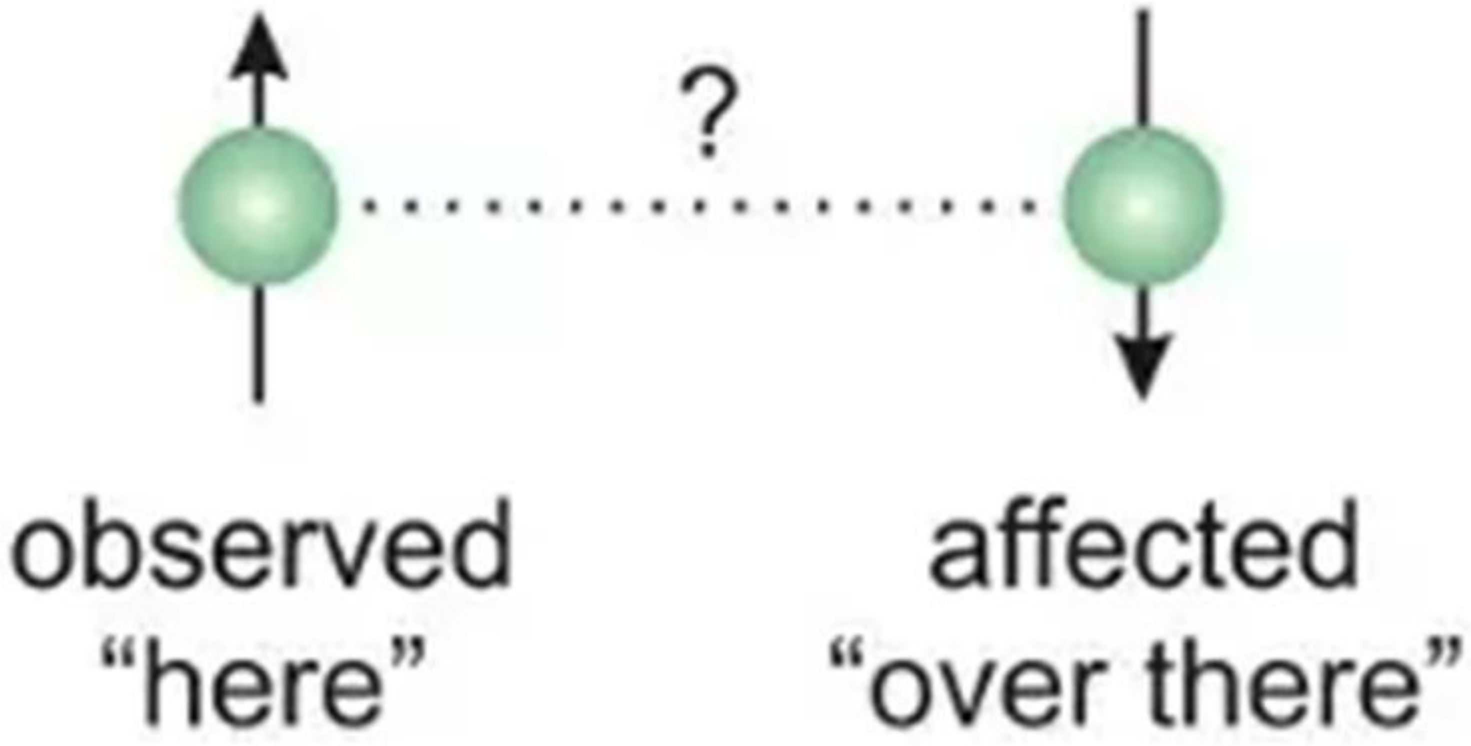
They interpreted as indicating that the explanation of physical reality provided by quantum mechanics was incomplete

Entanglement:

Spin state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$




?
observed
“here”
affected
“over there”

Question: Superluminal (v>c)?

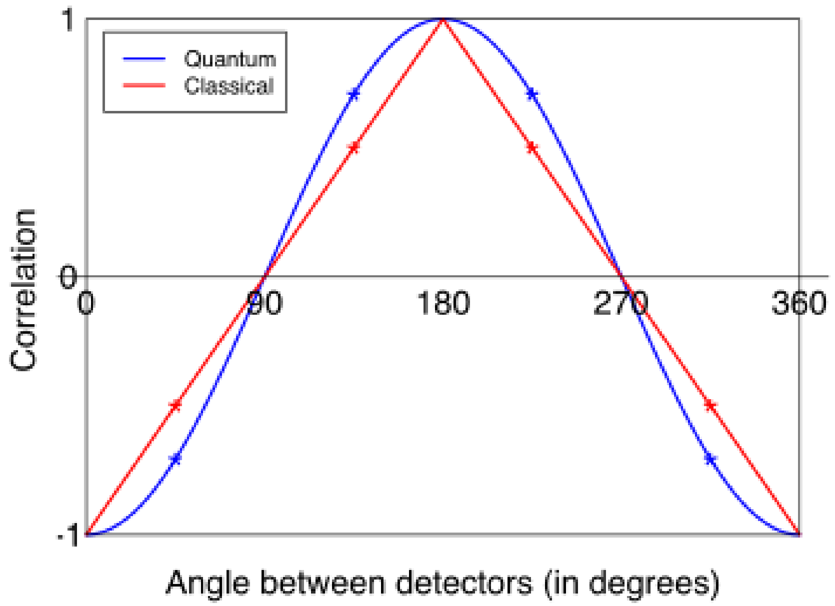
Bell's inequality

Bell's theorem, derived in his seminal 1964 paper titled On the Einstein Podolsky Rosen paradox.

Local realism

$$\mathrm{E}(X) = \int_\Lambda X(\lambda)p(\lambda)d\lambda,$$

$$C_h(a,c) - C_h(b,a) - C_h(b,c) \leq 1,$$



Violation of Bell inequality

nature > letters > article

MENU

nature

Letter | Published: 21 October 2015

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau & R. Hanson

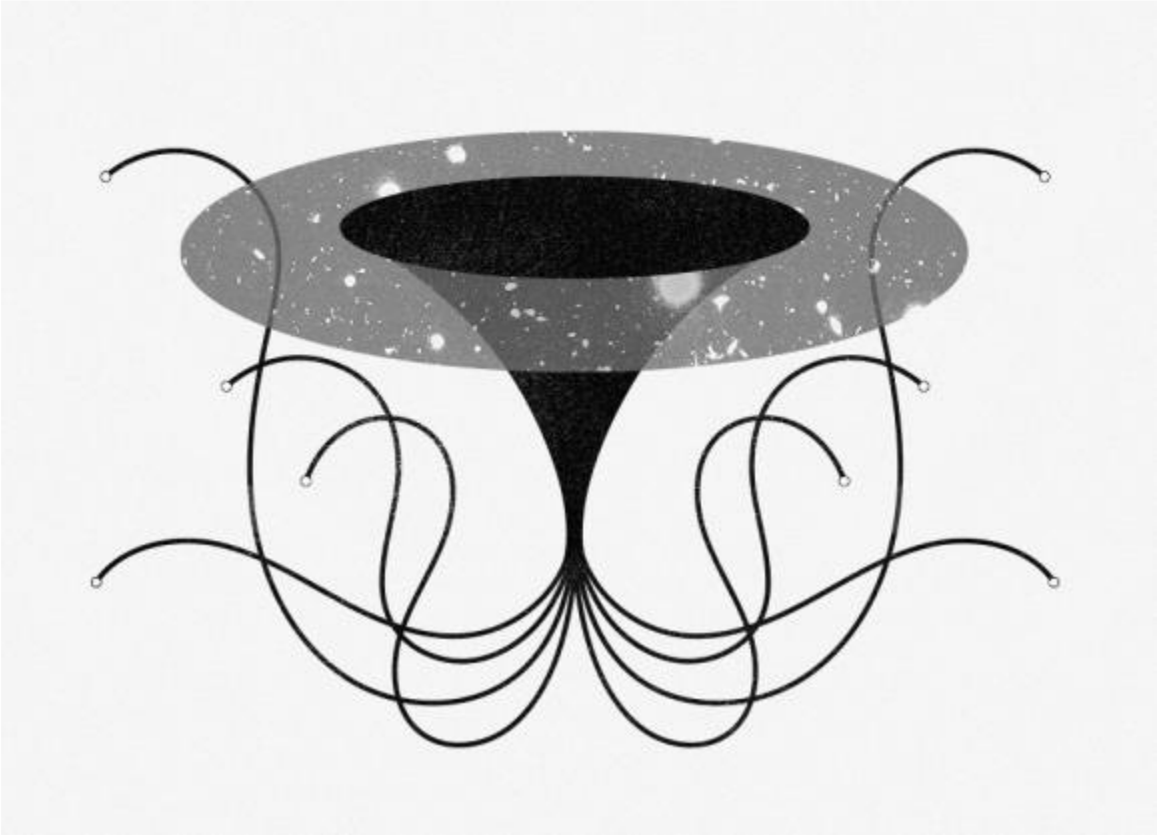
Nature **526**, 682–686(2015) | Cite this article

What is proved by impossibility proof is lack of imagination (Bell 1982, Holland's book)

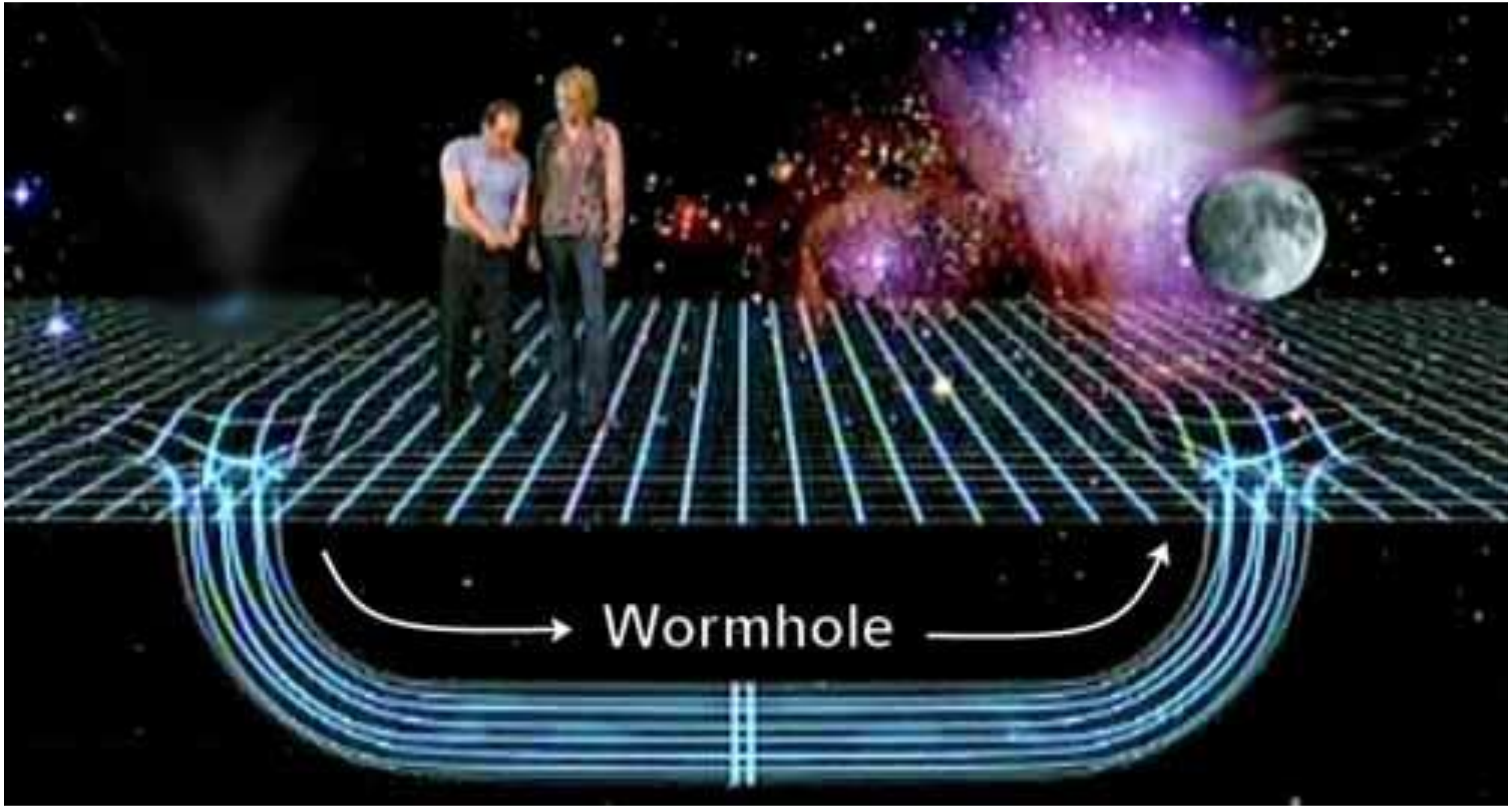
It perhaps cannot be stated with absolute certainty that Bell's theorem rules out forever the theoretical possibility of a local objective model that is compatible with all the quantum mechanical prediction in the type of experiments.

ER=EPR(Leonard Susskind and Juan Maldacena)

ER=EPR is a conjecture in physics stating that entangled particles are connected by a wormhole (or Einstein–Rosen bridge) and may be a basis for unifying general relativity and quantum mechanics into a theory of everything



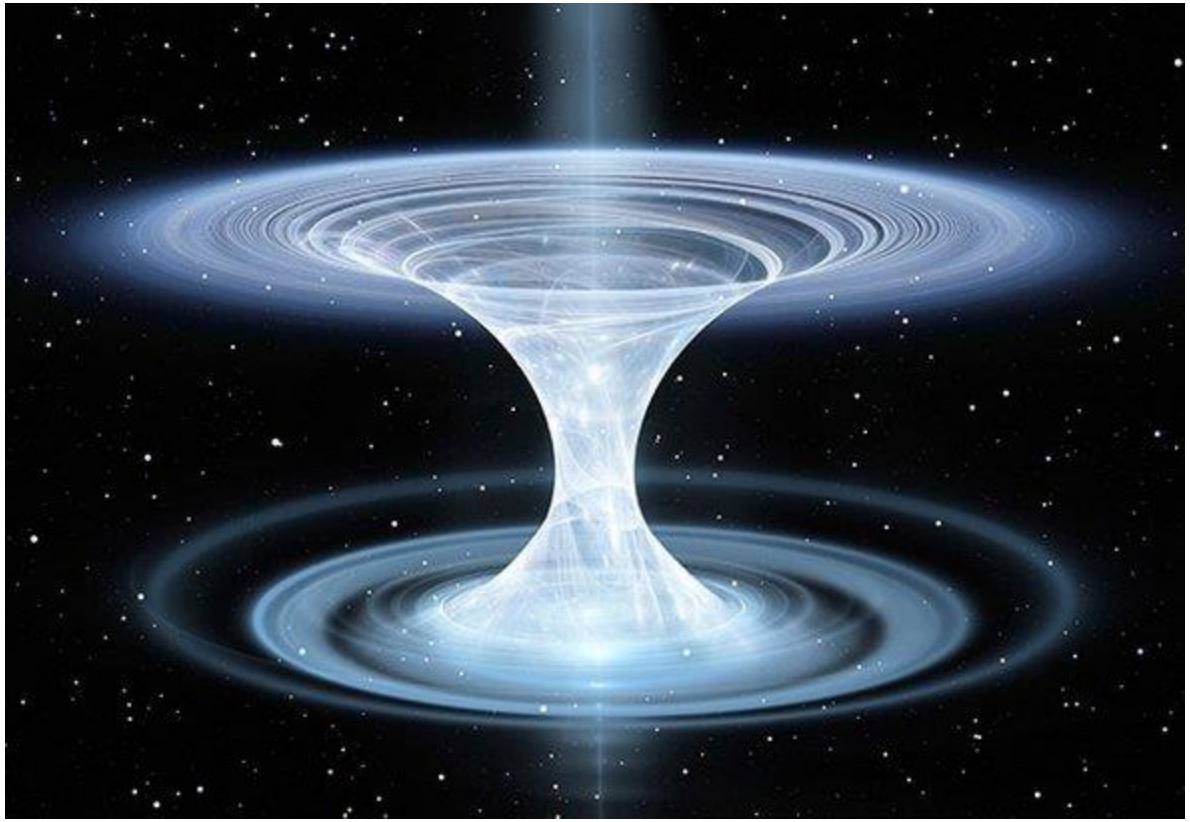
ER=EPR?



Wormhole connect two space points

ER=EPR?

Connected?



Metric?

Complete solution

$$d\tau^2 = -A(\lambda)dt^2 + B(\lambda)d\lambda^2 + r^2(\lambda)d\Omega,$$

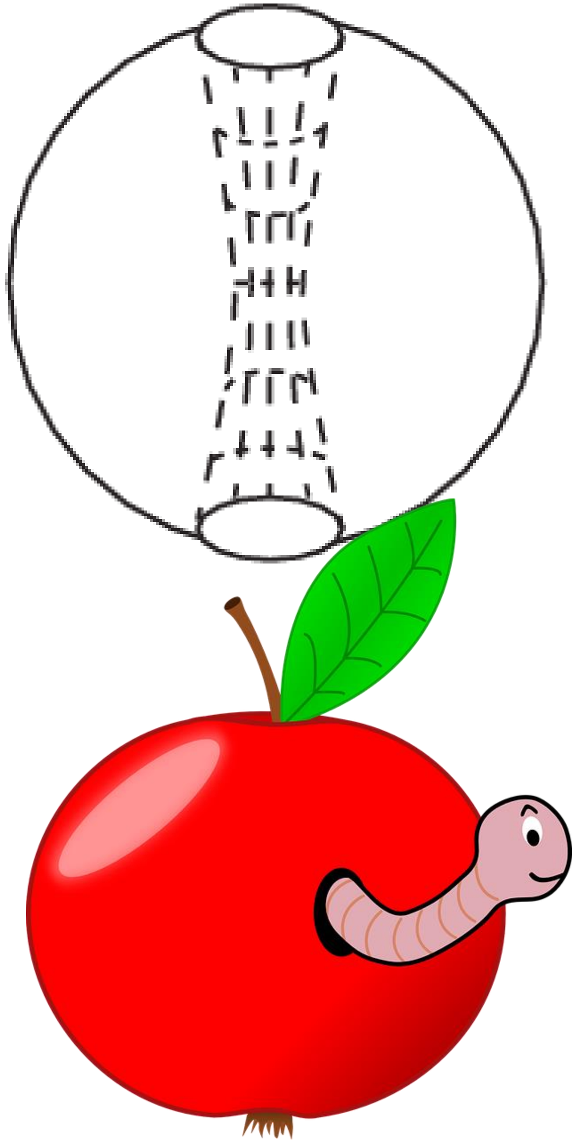
$$r = \frac{\cos(\alpha)}{\sqrt{3}} - \sin(\alpha),$$

$$\alpha = \frac{\arctan\left(\frac{\sqrt{3-81\,b^2}}{9\,b}\right)}{3},$$

$$b = M + \frac{a^2}{2}\sin^2\lambda,$$

$$A = 1 - \frac{2M}{r(\lambda)} - r^2(\lambda),$$

$$B = \frac{1}{1 - \frac{2\,M}{r(\lambda)} - r^2(\lambda)}\left(\frac{dr(\lambda)}{d\lambda}\right)^2.$$



PRD, 98, 124026

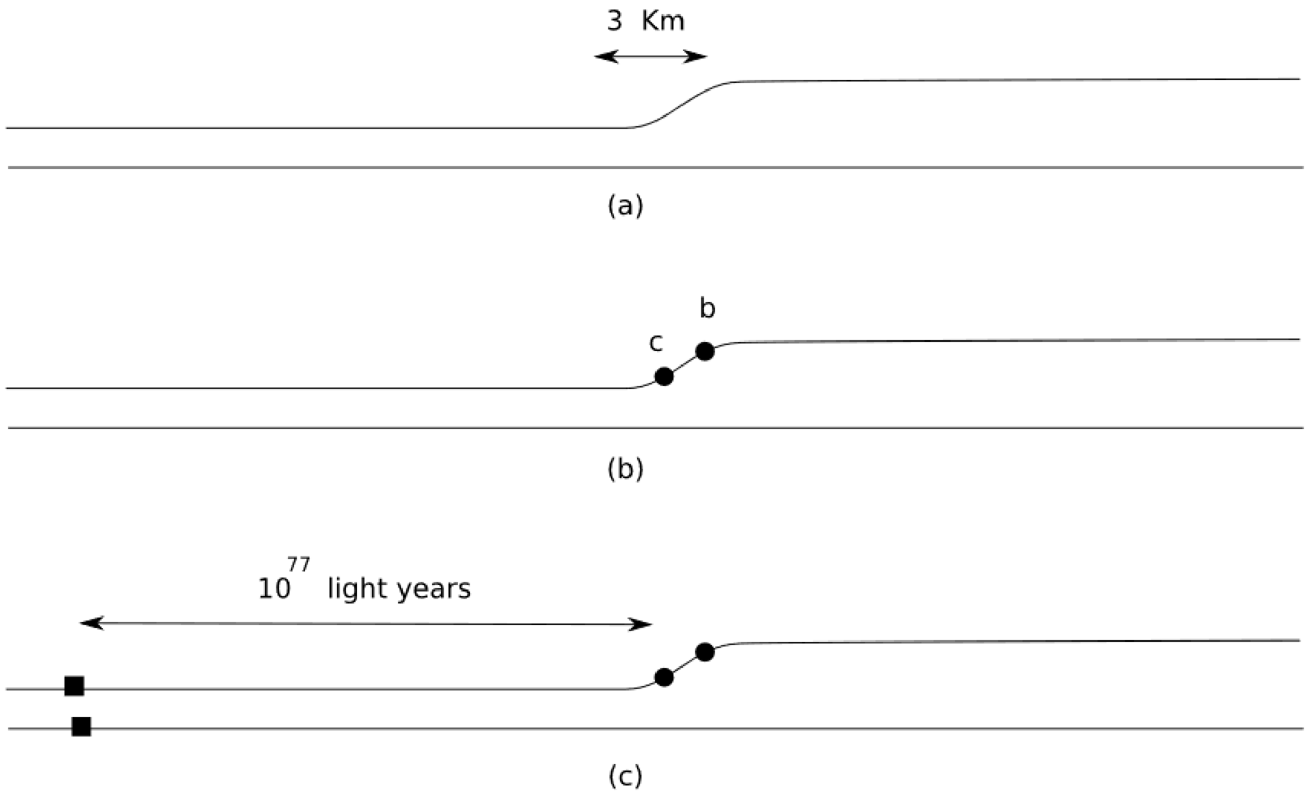
The information paradox: A pedagogical introduction

Class. Quant. Grav. 26, 224001

Samir D. Mathur

Pair creation

$$|\Psi\rangle_{\text{pair}} = \frac{1}{\sqrt{2}}|0\rangle_c|0\rangle_b + \frac{1}{\sqrt{2}}|1\rangle_c|1\rangle_b$$



Four Bell states

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

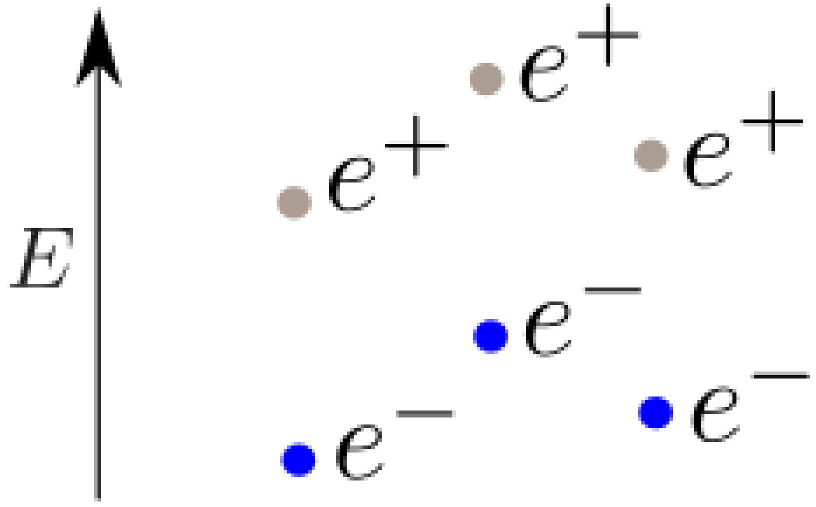
$$|\psi\rangle = \frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Schwinger pair production

Creation rate

$$\Gamma = \frac{(eE)^2}{4\pi^3 c\hbar^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \mathrm{e}^{-\frac{\pi m^2 c^3 n}{eE\hbar}}$$



Expected

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right).$$

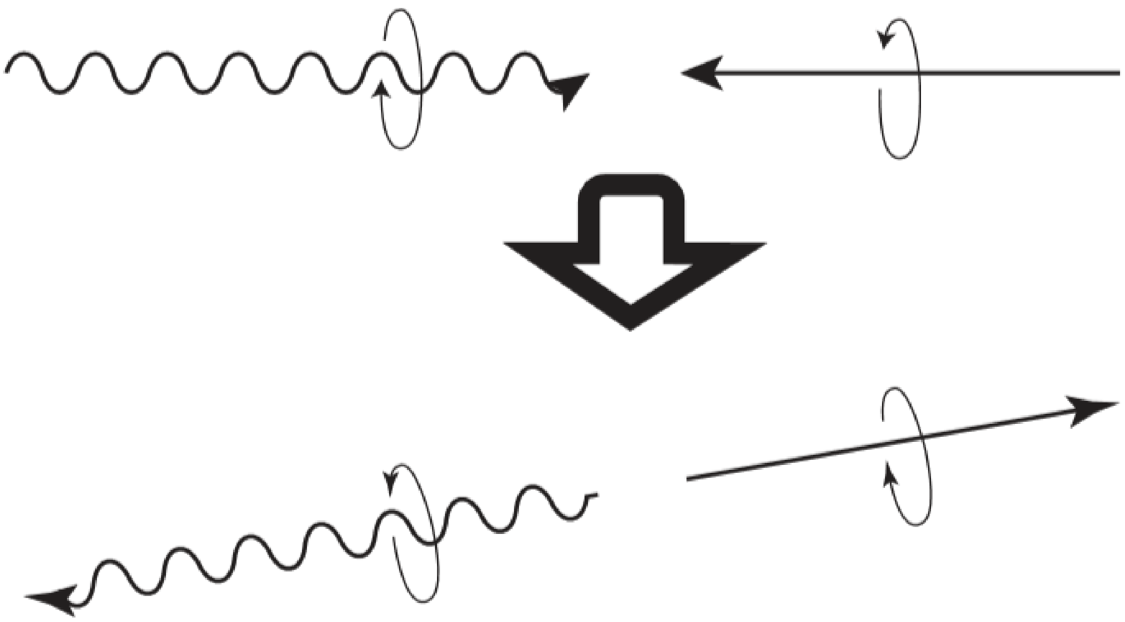
Example:

Y. Li, Y. Dai and Y. Shi, Phys. Rev. D 95, no. 3, 036006 (2017)

$$|0_{\mathbf{k}},0_{-\mathbf{k}}\rangle^{\mathrm{in}} = x_0|0_{\mathbf{k}},0_{-\mathbf{k}}\rangle^{\mathrm{out}} + x_1|\uparrow_{\mathbf{k}},\downarrow_{-\mathbf{k}}\rangle^{\mathrm{out}} + x_2|\downarrow_{\mathbf{k}},\uparrow_{-\mathbf{k}}\rangle^{\mathrm{out}} + x_3|\uparrow\downarrow_{\mathbf{k}},\uparrow\downarrow_{-\mathbf{k}}\rangle^{\mathrm{out}},$$

Compton scattering in the center of mass frame

A right handed photon and a right handed electron collide and turn almost completely backward. The reflected photon and electron are both right handed. The total spin appears not to be conserved because there is angular momentum involved in the process.



Why it is not

$$A_0 \left|\uparrow\downarrow\right\rangle + A_1 \left|\downarrow\uparrow\right\rangle + A_2 \left|\uparrow\uparrow\right\rangle + A_3 \left|\downarrow\downarrow\right\rangle .$$

$$\overset{?}{\Longleftrightarrow}$$

$$\frac{1}{\sqrt{2}}(\left|\uparrow\downarrow\right\rangle \pm \left|\downarrow\uparrow\right\rangle).$$

The Lagrangian density for electrodynamics

follow Klunger et al.'s study

$$L = \bar{\psi} i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The γ matrices

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}$$

The equation of motion

$$(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m)\psi = 0.$$

ψ can be expressed through a new field ϕ as

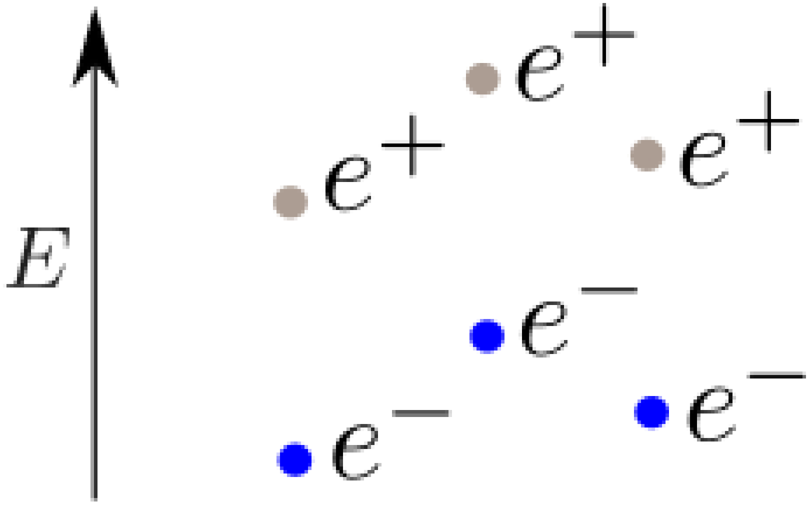
$$\psi = (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + m)\phi.$$

The equation of motion becomes

$$\left[(i\partial_\mu - eA_\mu)^2 - \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\right]\phi = 0.$$

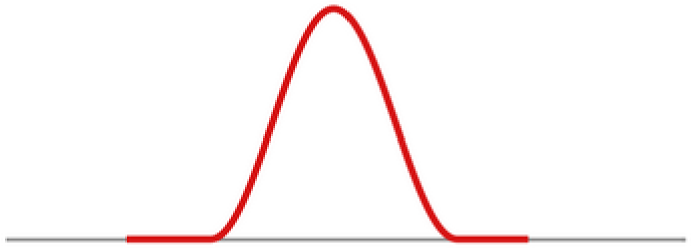
Spatial homogeneity

The none-zero A_μ component is $A_3 = a(t)$.



$$a(-\infty) = 0.$$

$$\lim_{t\to\infty} a(t) = \text{constant}.$$



$$\left[\partial_\mu \partial^\mu + e^2 a^2 + 2ia\partial_3 - ie\partial_0 a \gamma^0 \gamma^3 + m^2\right]\phi = 0.$$

$$\phi_{\boldsymbol{k},j} = e^{i\boldsymbol{k}\cdot\boldsymbol{x}} f_{\boldsymbol{k},j}\chi_j,$$

$$\chi_1 = \begin{bmatrix}\eta^1\\ \eta^1\end{bmatrix}, \chi_2 = \begin{bmatrix}\eta^2\\ -\eta^2\end{bmatrix}, \eta^1 = \begin{bmatrix}1\\ 0\end{bmatrix}, \eta^2 = \begin{bmatrix}0\\ 1\end{bmatrix}$$

These spinors are the eigenvectors of γ0γ3 in the representation of γ matrices. They are not exactly the same as the spin up and spin down eigen vectors.

$$\sum_{\alpha=1}^{4} (\chi_r^\dagger)^\alpha (\chi_s)_\alpha = 2\delta_{rs}.$$

The mode function $f_{\boldsymbol{k},j}$ satisfies

$$\frac{d^2 f_{\boldsymbol{k},j}}{dt^2} + \left(\omega_{\boldsymbol{k}}^2 - ie\frac{da}{dt}\right) f_{\boldsymbol{k},j} = 0.$$

$$\omega_{\boldsymbol{k}}^2 = p_3^2 + \boldsymbol{k}_-^2 + m^2, \ \boldsymbol{k}_-^2 = k_1^2 + k_2^2$$

$$p_i = k^i - eA^{'i}.$$

$$\psi^{\pm}_{\boldsymbol{k},j} = (i\gamma^0\partial_0 + \gamma^i k_i - e\gamma^3 A_3 + m)\phi^{\pm}_{\boldsymbol{k},j}$$

Normalization

$$\psi^{\pm\dagger}_{\boldsymbol{r}}\psi^{\pm}_{\boldsymbol{s}} = \delta_{\boldsymbol{rs}},\ \psi^{\pm\dagger}_{\boldsymbol{r}}\psi^{\mp}_{\boldsymbol{s}} = 0.$$

$$\psi = \int \sum_{j=1,2} \left[b_j(\boldsymbol{k}) \psi^+_{\boldsymbol{k},j} + d^\dagger_j(-\boldsymbol{k}) \psi^-_{\boldsymbol{k},j} \right] \frac{d\boldsymbol{k}}{(2\pi)^3}$$

$$\{b_r(\boldsymbol{k}), b^\dagger_s(\boldsymbol{q})\} = \{d_r(\boldsymbol{k}), d^\dagger_s(\boldsymbol{q})\} = (2\pi)^3 \delta^3(\boldsymbol{k} - \boldsymbol{q}) \delta_{rs}$$

$$\{\psi_\alpha(t, \boldsymbol{x}), \psi^\dagger_\beta(t, \boldsymbol{y}))\} = \delta^3(\boldsymbol{x} - \boldsymbol{y}) \delta_{\alpha\beta}.$$

"IN" VACUUM

$$b_s(\boldsymbol{k}) \left|0, \mathrm{in}\right\rangle = d_s(-\boldsymbol{k}) \left|0, \mathrm{in}\right\rangle = 0$$

change the representation to up and down spinor bases,

$$\psi = \sum_{r=1,2} \int \Big[b_r^{(0)}(\boldsymbol{k},t) u_{r,\boldsymbol{k}} e^{-i\int \omega_{\boldsymbol{k}} dt} + d_r^{(0)\dagger}(-\boldsymbol{k},t) v_{r,-\boldsymbol{k}} e^{i\int \omega_{\boldsymbol{k}} dt} \Big] e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \frac{d\boldsymbol{k}}{(2\pi)^3}.$$

Here, $u_{r,\boldsymbol{k}}$ and $v_{r,-\boldsymbol{k}}$ are defined as

$$u_{r,\boldsymbol{k}} = \begin{bmatrix} \sqrt{\frac{\omega_{\boldsymbol{k}}+m}{2\omega_{\boldsymbol{k}}}}\eta^r \\ \frac{\vec{\sigma}\cdot\vec{p}}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}}+m)}}\eta^r \end{bmatrix}, \ v_{r,-\boldsymbol{k}} = \begin{bmatrix} \frac{-\vec{\sigma}\cdot\vec{p}}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}}+m)}}\eta^r \\ \sqrt{\frac{\omega_{\boldsymbol{k}}+m}{2\omega_{\boldsymbol{k}}}}\eta^r \end{bmatrix}$$

where $u_{1,k}$ and $u_{2,k}$ are spin up and spin down electron spinor respectively (along z-direction). $v_{1,-k}$ and $v_{2,-k}$ are spin down and spin up positron spinors respectively.

$$u^{\dagger}_{r,\boldsymbol{k}} u_{r',\boldsymbol{k}} = \delta_{r,r'}, \ v^{\dagger}_{r,\boldsymbol{k}} v_{r',\boldsymbol{k}} = \delta_{r,r'}, \ u^{\dagger}_{r,\boldsymbol{k}} v_{r',-\boldsymbol{k}} = 0.$$

“OUT” VACUUM.

$$b_r^{(0)}(\boldsymbol{k},t)\,|0,\mathrm{out}\rangle = d_r^{(0)}(-\boldsymbol{k},t)\,|0,\mathrm{out}\rangle = 0$$

Bogoliubov transformation

$$
\begin{aligned}
b_r^{(0)}(\boldsymbol{k},t) &= \sum_{s=1,2} \alpha^s_{\boldsymbol{k},r}(t) b_s(\boldsymbol{k}) + \beta^s_{\boldsymbol{k},r}(t) d_s(-\boldsymbol{k})^\dagger \\
d_r^{(0)}(-\boldsymbol{k},t)^\dagger &= \sum_{s=1,2} -\beta^{*s}_{\boldsymbol{k},r}(t) b_s(\boldsymbol{k}) + \alpha^{*s}_{\boldsymbol{k},r}(t) d_s(-\boldsymbol{k})
\end{aligned}
$$

From the canonical anti-communication relation

$$\sum_{r=1,2} (|\alpha^{s}_{\boldsymbol{k},r}|^2 + |\beta^{s}_{\boldsymbol{k},r}|^2) = 1$$

$\psi^{+}_{\boldsymbol{k},s}$ and $\psi^{-}_{\boldsymbol{k},s}$ are found

$$\begin{aligned}
\psi^{+}_{\boldsymbol{k},s} &= \sum_{r=1,2} \alpha^{s}_{\boldsymbol{k},r} u_{r,\boldsymbol{k}} e^{-i\int \omega_{\boldsymbol{k}} dt} - \beta^{*s}_{\boldsymbol{k},r} v_{r,-\boldsymbol{k}} e^{i\int \omega_{\boldsymbol{k}} dt} \\
\psi^{-}_{\boldsymbol{k},s} &= \sum_{r=1,2} \beta^{s}_{\boldsymbol{k},r} u_{r,\boldsymbol{k}} e^{-i\int \omega_{\boldsymbol{k}} dt} + \alpha^{*s}_{\boldsymbol{k},r} v_{r,-\boldsymbol{k}} e^{i\int \omega_{\boldsymbol{k}} dt}
\end{aligned}$$

$$
\begin{aligned}
\beta^{*1}_{\boldsymbol{k},1} &= -e^{-i\int \omega_{\boldsymbol{k}} dt} \frac{(\omega_{\boldsymbol{k}} + m + p_3)(\omega_{\boldsymbol{k}} f^{+}_{\boldsymbol{k},1} - i\dot{f}^{+}_{\boldsymbol{k},1})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}} \\
\beta^{*1}_{\boldsymbol{k},2} &= -e^{-i\int \omega_{\boldsymbol{k}} dt} \frac{(p_1 + ip_2)(\omega_{\boldsymbol{k}} f^{+}_{\boldsymbol{k},1} - i\dot{f}^{+}_{\boldsymbol{k},1})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}} \\
\alpha^{1}_{\boldsymbol{k},1} &= e^{i\int \omega_{\boldsymbol{k}} dt} \frac{(\omega_{\boldsymbol{k}} + m - p_3)(\omega_{\boldsymbol{k}} f^{+}_{\boldsymbol{k},1} + i\dot{f}^{+}_{\boldsymbol{k},1})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}} \\
\alpha^{1}_{\boldsymbol{k},2} &= e^{i\int \omega_{\boldsymbol{k}} dt} \frac{-(p_1 + ip_2)(\omega_{\boldsymbol{k}} f^{+}_{\boldsymbol{k},1} + i\dot{f}^{+}_{\boldsymbol{k},1})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}}
\end{aligned}
$$

$$\beta^{*2}_{\boldsymbol{k},1} = -e^{-i\int \omega_{\boldsymbol{k}} dt} \frac{(p_1 - ip_2)(\omega_{\boldsymbol{k}} f^+_{\boldsymbol{k},2} - i\dot{f}^+_{\boldsymbol{k},2})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}}$$

$$\beta^{*2}_{\boldsymbol{k},2} = e^{-i\int \omega_{\boldsymbol{k}} dt} \frac{(\omega_{\boldsymbol{k}} + m + p_3)(\omega_{\boldsymbol{k}} f^+_{\boldsymbol{k},2} - i\dot{f}^+_{\boldsymbol{k},2})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}}$$

$$\alpha^{2}_{\boldsymbol{k},1} = e^{i\int \omega_{\boldsymbol{k}} dt} \frac{(p_1 - ip_2)(\omega_{\boldsymbol{k}} f^+_{\boldsymbol{k},2} + i\dot{f}^+_{\boldsymbol{k},2})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}}$$

$$\alpha^{2}_{\boldsymbol{k},2} = e^{i\int \omega_{\boldsymbol{k}} dt} \frac{(\omega_{\boldsymbol{k}} + m - p_3)(\omega_{\boldsymbol{k}} f^+_{\boldsymbol{k},2} + i\dot{f}^+_{\boldsymbol{k},2})}{\sqrt{2\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)}}$$

The number of particles produced per unit phase space volume at a given momentum is given by

$$
\begin{aligned}
n(\boldsymbol{k}, t) &= \sum_{r=1,2} \langle 0, \mathrm{in}|\, b_{\boldsymbol{r}}^{(0)\dagger}(\boldsymbol{k}, t) b_{\boldsymbol{r}}^{(0)}(\boldsymbol{k}, t)\, |0, \mathrm{in}\rangle \\
&= \sum_{s=1,2;r=1,2} |\beta_{\boldsymbol{k},r}^{s}(t)|^2
\end{aligned}
$$

The amplitude

$$T_{rs} = \langle \text{out}| \, b_r^{(0)}(\boldsymbol{k}, t) d_s^{(0)}(-\boldsymbol{k}, t) \, |\text{in}\rangle$$
$$= A \sum_{i=1,2} \beta^i_{\boldsymbol{k},r}(t) \Big(\alpha^i_{\boldsymbol{k},s}(t) - \sum_{j=1,2} \beta^j_{\boldsymbol{k},s}(t) B^*_{ij} \Big)$$

Here

$$|0, \text{out}\rangle = \prod_{\boldsymbol{k},s} A \exp(\sum_{ij} B_{ij} b_i^\dagger d_j^\dagger) \, |0, \text{in}\rangle$$

$$B_{ij} = (-1)^m \frac{\alpha^m_{\boldsymbol{k},2}(t)\beta^j_{\boldsymbol{k},1}(t) - \alpha^m_{\boldsymbol{k},1}(t)\beta^j_{\boldsymbol{k},2}(t)}{\alpha^2_{\boldsymbol{k},2}(t)\alpha^1_{\boldsymbol{k},1}(t) - \alpha^2_{\boldsymbol{k},1}(t)\alpha^1_{\boldsymbol{k},2}(t)}$$
$$A = \sqrt{1 + (|B_{11}| + |B_{12}| + |B_{21}| + |B_{22}|)^2}.$$

Final result

$$T_{11} = -\frac{\omega_{\boldsymbol{k}}^2 + m\omega_{\boldsymbol{k}} - p_3^2}{\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)} E + O(\beta^3) \qquad |\uparrow\downarrow\rangle$$

$$T_{12} = \frac{p_3(p_1 + ip_2)}{\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)} E + O(\beta^3) \qquad |\uparrow\uparrow\rangle$$

$$T_{21} = \frac{p_3(p_1 - ip_2)}{\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)} E + O(\beta^3) \qquad |\downarrow\downarrow\rangle$$

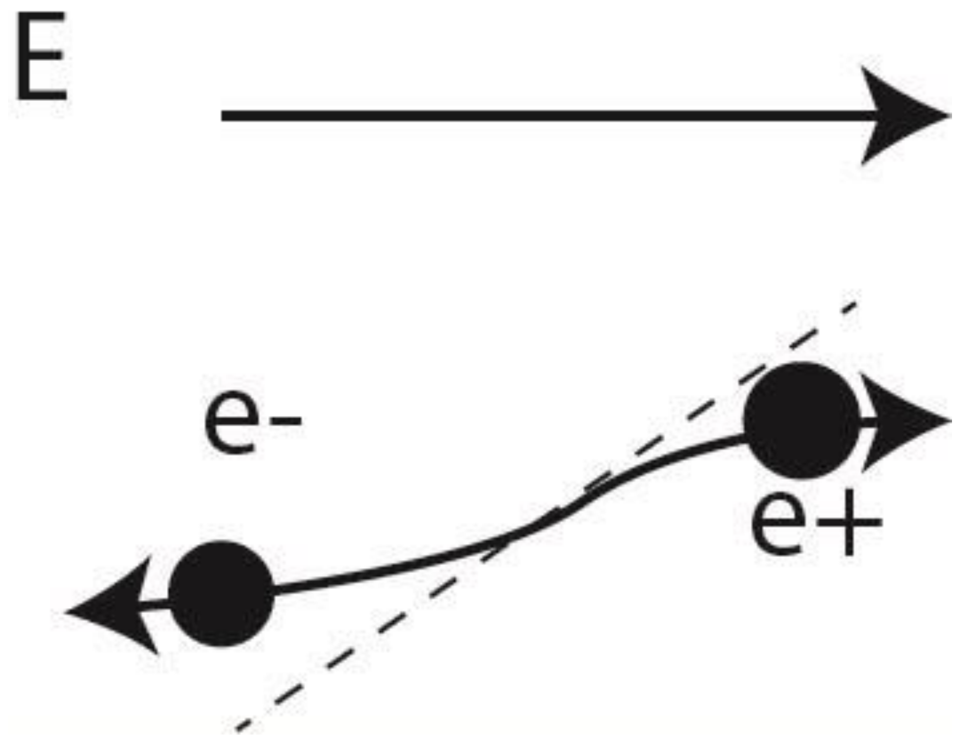
$$T_{22} = \frac{\omega_{\boldsymbol{k}}^2 + m\omega_{\boldsymbol{k}} - p_3^2}{\omega_{\boldsymbol{k}}(\omega_{\boldsymbol{k}} + m)} E + O(\beta^3) \qquad |\downarrow\uparrow\rangle$$

$$E = (\omega_{\boldsymbol{k}} \bar{f}^{+}_{\boldsymbol{k},1} + i\dot{\bar{f}}^{+}_{\boldsymbol{k},1})(\omega_{\boldsymbol{k}} f^{+}_{\boldsymbol{k},1} + i\dot{f}^{+}_{\boldsymbol{k},1}) e^{2i\int \omega_{\boldsymbol{k}} dt} A$$

It is not always a Bell's state

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle).$$

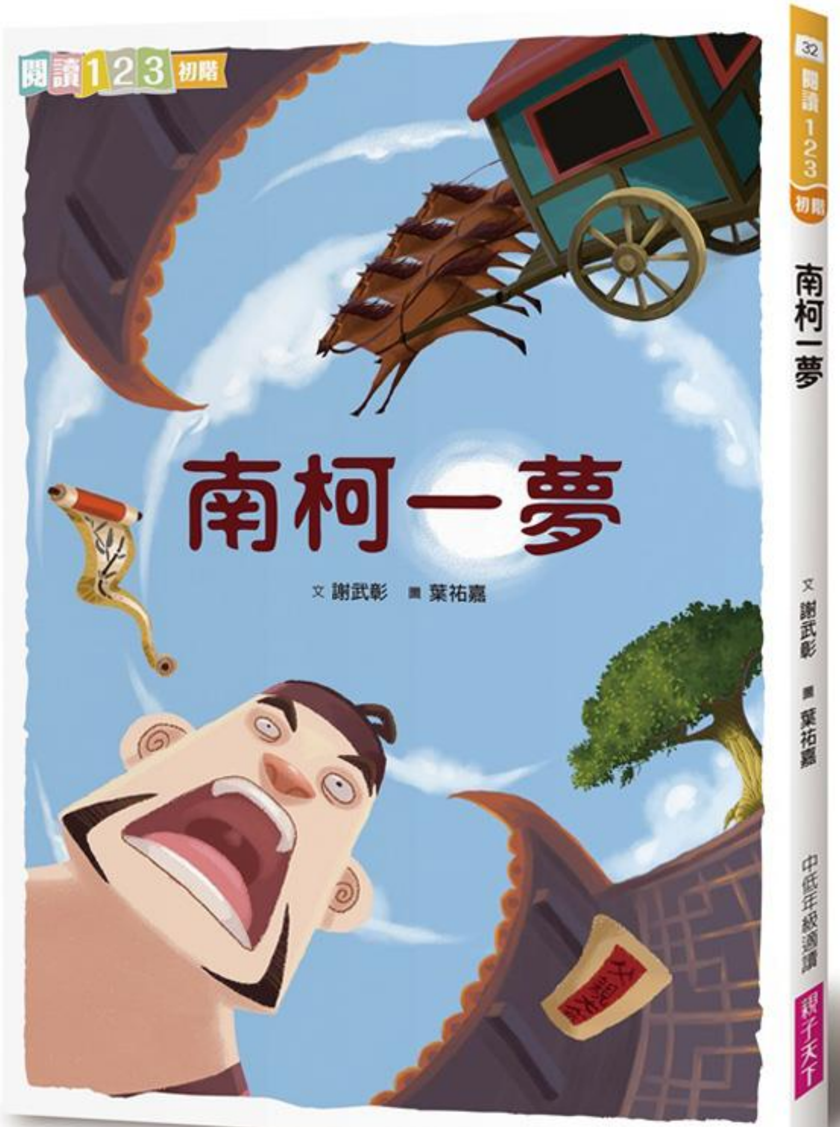
Angular momentum is involved



Conclusion:

- **A state of a particle pair produced by the Schwinger effect is not necessarily a maximally entangled Bell's state**
- **The argument of information paradox may be too simple.**

An empty dream



All have a good trip